# Some Unsolved Crux Problems from 2000

**2558**. *Proposed by Peter Y. Woo, Biola University, La Mirada, CA, USA.*

Let $Z$ be a half-plane bounded by a line $L$. Let $A$, $B$ and $C$ be any three points on $L$ such that $C$ lies between $A$ and $B$. Denote the three semicircles in $Z$ on $AB$, $AC$ and $CB$ as diameters by $K_0$, $K_1$ and $K_2$ respectively. Let $F$ be the family of semicircles in $Z$ with diameters on $L$ (including all half-lines in $Z$ perpendicular to $L$). Denote by $f_{XY}$ the unique semicircle passing through the pair of distinct points $X$, $Y$ in $Z \cup L$. Let $P$, $Q$, $R$, be three points on $K_2$, $K_1$, $K_0$, respectively.

If $f_{AP}$, $f_{BQ}$ and $f_{CR}$ concur at $T$, and the lines $AP$, $BQ$, $CR$ concur at $S$, prove that $f_{AP}$, $f_{BQ}$ and $f_{CR}$ are orthogonal to $K_2$, $K_1$, $K_0$, respectively, and that the circle $PQR$ is tangent to each semicircle $K_j$, $(j = 0, 1, 2)$.

**2561**. *Proposed by Hassan A. ShahAli, Tehran, Iran.*

Let $M$ disks from $N$ different colours be placed in a row such that $k_i$ disks are from the $i^{th}$ colour $(i = 1, 2, \ldots, N)$ and $k_1 + k_2 + \cdots + k_N = M$.

A move is an exchange of two adjacent disks.

Determine the smallest number of moves needed to rearrange the row such that all disks of the same colour are adjacent to one another.

**2593**. *Proposed by Nairi M. Sedrakyan, Yerevan, Armenia.*

Let $S(a)$ denote the sum of the digits of the natural number $a$. Let $k$ and $n$ be natural numbers with $(n, 3) = 1$. Prove that there exists a natural number $m$ which is divisible by $n$ and $S(m) = k$ if either

(a) $k > n - 2$; or

(b) $k > S^2(n) + 7S(n) - 9$.